

Low-temperature behavior of a Magnetic Impurity in a Heisenberg Chain

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Abstract

Using the bosonization technique, we have studied a spin-1/2 magnetic impurity in Heisenberg chain, and shown that the impurity specific heat and spin susceptibility have an anomalous temperature dependence. The temperature dependence of the impurity specific heat is: $C_{im}(T) \sim T^\mu$, $\mu = 4 - \frac{1}{g}(1-g)^2$, for $g_c < g < 1$. The impurity spin susceptibility has the following temperature dependence: $\chi_{im}(T) \sim T^\nu$, $\nu = 3 - \frac{1}{g}(1-g)^2$, for $g_c < g < 1$, and $\nu = -1$, for $g \leq g_c$, where g_c satisfies: $4g_c = (1 - g_c)^2$, and g is a dimensionless coupling strength parameter.

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Recently, the quantum impurity scattering of the Tomonaga-Luttinger(TL) liquid has been extensively studied by using different techniques [1–19]. There is some controversy on the treatment of backward scattering of the conduction electrons on a quantum impurity or impurity-like hole in the valence band. One thinks that including backward scattering drastically changes the properties of a TL-liquid. In principle, this problem can be formulated by means of the Bethe-Ansatz to obtain exact results for static and thermodynamic quantities. However to obtain frequency dependent quantities one has to use perturbative methods. Due to strong correlations between the conduction electrons and impurity, perturbation-theory may fail. In Ref. [19], we have given a clear expression and some exact results within the validity of the bosonization method for a quantum impurity scattering of a general one-dimensional(1D) interacting electronic system, and shown that the system has two independent collective modes: one is drastically influenced by electron-electron interaction, and another is independent of the electron-electron interaction which is clearly observed in the numerical calculations [18], while it is missed in the renormalization group calculations [3]. Here we use the same method as that in Ref. [19] to study a magnetic impurity in the Heisenberg chain which can be easily treated by numerical methods. The impurity susceptibility of this system shows an unusual temperature dependence: $\chi_{im}(T) \sim T^\nu$, $\nu = 3 - \frac{1}{g}(1 - g)^2$, for $g_c < g < 1$, and $\nu = -1$, for $g \leq g_c$, where g_c is defined as that: $4g_c = (1 - g_c)^2$, and g is a dimensionless coupling strength parameter. For an antiferromagnetic Heisenberg chain, the dimensionless coupling strength parameter g takes the value [21] $g = \frac{1}{2}$, and the temperature dependence of the impurity susceptibility $\chi_{im}(T)$ is $T^{5/2}$. The impurity specific heat $C_{im}(T)$ has the temperature dependence: $C_{im}(T) \sim T^\mu$, $\mu = 4 - \frac{1}{g}(1 - g)^2$, for $g_c < g < 1$. Although the impurity specific heat as well as the spin susceptibility depend on the dimensionless coupling strength parameter g , we can still define a temperature independent Wilson-ratio in this system.

We consider the following one-dimensional Heisenberg model

$$H = - \sum_i \left[J_\perp (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y) + J_z S_i^z S_{i+1}^z \right] \quad (1)$$

where J_\perp is the transverse exchange interaction strength, and J_z the longitudinal exchange interaction strength. Using the Jordan-Wigner transformation to spinless fermion operators f_i , $S_i^+ = S_i^x + iS_i^y = f_i^+ \exp \left[-i\pi \sum_{l < i} n_l^f \right]$, $S_i^z = f_i^+ f_i - \frac{1}{2}$ and after a Fourier-transformation, the Hamiltonian can be written in the form

$$H = -J_\perp \sum_k \cos(ak) f_k^+ f_k - \frac{J_z}{N} \sum_k \cos(ak) \rho(k) \rho(-k) \quad (2)$$

where a is the lattice constant, N is the site number, the sum over k is restricted to the first Brillouin zone, $f_k = \frac{1}{\sqrt{N}} \sum_j e^{ikx_j} f_j$, and $\rho(k)$ is the density operator, $\rho(k) = \sum_j e^{ikx_j} f_j^+ f_j$. One expects the asymptotic behavior of correlation functions to be determined by the low-lying excited states near the Fermi points at $ak_F = \pm \frac{\pi}{2}$. Therefore one can consider a related model with two linear single-particle spectra tangent to the $-\cos(ak)$ at the Fermi points. We may introduce $f_{1k}(f_{2k})$ operators to describe the fermion particles with positive (negative) group velocity and the associated fields, $\psi_i(x) = \frac{1}{\sqrt{L}} \sum_k f_{ik} e^{ikx}$, $i = 1, 2$, where $L = aN$ is the length of the system. The density operator is $\rho(k) = \rho_1(k) + \rho_2(k)$ with $\rho_i(k) = \sum_p f_{ik+p}^+ f_{ip}$ for $i = 1, 2$. For the case of spinless fermions, the J_z -term in Eq. (2) is $\sum_{k \sim 2k_F} \rho(k) \rho(-k) \rightarrow -\sum_{k \sim 0} \rho_1(k) \rho_2(-k)$. Therefore the Hamiltonian (2) can be simplified as

$$H = v_F \sum_k k (f_{1k}^+ f_{1k} - f_{2k}^+ f_{2k}) - \frac{4J_z}{N} \sum_k \rho_1(k) \rho_2(-k). \quad (3)$$

The Jordan-Wigner transformation for the spin operators on the lattice has the obvious generalization to the continuum situation [21]

$$f_i \rightarrow \left(\frac{a}{2}\right)^{1/2} [\psi_1(x) + \psi_2(x)] \quad (4)$$

$$i\pi \sum_{l < i} f_l^+ f_l \rightarrow i\pi \int_{-\infty}^{x_i - a} dy [\rho_1(y) + \rho_2(y) + (2a)^{-1}] \equiv iN(x_i)$$

and the resulting representation for continuum spin operators is

$$S^-(x) = \left(\frac{1}{2a}\right)^{1/2} [\psi_1(x) + \psi_2(x)] e^{iN(x)}$$

$$S^+(x) = [S^-(x)]^+ \quad (5)$$

$$2S^z(x) = \rho_1(x) + \rho_2(x) + \psi_1^+(x) \psi_2(x) + \psi_2^+(x) \psi_1(x)$$

These representations of the spin operators are different from that used in Ref. [20]. Different results of Ref. [20] may come from the special choice for the spin operators and the Kondo interaction between the impurity and the conduction electrons. It is well-known that the bosonization representations of the fermion fields are [21–23]

$$\begin{aligned}\psi_i(x) &= \frac{e^{\pm ik_F x}}{\sqrt{2\pi\alpha}} e^{i\Phi_i(x)} \\ \Phi_i(x) &= \mp i \frac{2\pi}{L} \sum_k \frac{e^{-\frac{\alpha}{2}|k| - ikx}}{k} \rho_i(k)\end{aligned}\quad (6)$$

where the negative sign for $i = 1$ and the positive sign for $i = 2$. α^{-1} is an ultraviolet cutoff, which is of the order of the conduction band width. The Hamiltonian (3) can be written in terms of boson density operators

$$\begin{aligned}H &= \frac{2\pi v_F}{L} \sum_{k>0} [\rho_1(-k)\rho_1(k) + \rho_2(-k)\rho_2(k)] \\ &\quad - \frac{4J_z}{L} \sum_k \rho_1(k)\rho_2(-k) \\ &= \frac{v_F}{8\pi} (1 - \gamma^2)^{\frac{1}{2}} \int dx [(\partial_x \tilde{\Phi}_+(x))^2 + (\partial_x \tilde{\Phi}_-(x))^2]\end{aligned}\quad (7)$$

where $[\rho_1(-k), \rho_1(k')] = [\rho_2(k'), \rho_2(-k)] = \frac{kL}{2\pi} \delta_{kk'}$; $\Phi_{\pm}(x) = \Phi_1(x) \pm \Phi_2(x)$, $\tilde{\Phi}_+(x) = (\frac{1+\gamma}{1-\gamma})^{1/4} \Phi_+(x)$, $\tilde{\Phi}_-(x) = (\frac{1-\gamma}{1+\gamma})^{1/4} \Phi_-(x)$, $\partial_x \Phi_{\pm}(x) = 2\pi(\rho_1(x) \pm \rho_2(x))$, $\gamma = -2J_z/(\pi v_F)$. The boson fields $\tilde{\Phi}_{\pm}$ are conjugate variables, $[\partial_x \tilde{\Phi}_+, \tilde{\Phi}_-(y)] = i4\pi\delta(x-y)$, $[\partial_x \tilde{\Phi}_-(x), \tilde{\Phi}_+(y)] = i4\pi\delta(x-y)$.

We consider a spin-1/2 magnetic impurity scattering on the conduction fermions

$$H_{im} = \tilde{J}_0^{\perp} (S^+ S^-(0) + S^- S^+(0)) + \tilde{J}_0^z S^z S^z(0) \quad (8)$$

By taking $S^+ = \tilde{f}^+$, $S^- = \tilde{f}$, $S^z = \tilde{f}^+ \tilde{f} - \frac{1}{2}$, the Hamiltonian (8) can be written as

$$\begin{aligned}H_{im} &= J_0^{\perp} [\tilde{f}^+ (\psi_1(0) + \psi_2(0)) e^{iN(0)} + h.c.] \\ &\quad + J_0^z (\tilde{f}^+ \tilde{f} - \frac{1}{2}) [\rho_1(0) + \rho_2(0) + \psi_1^{\dagger}(0) \psi_2(0) + \psi_2^{\dagger}(0) \psi_1(0)]\end{aligned}\quad (9)$$

where $J_0^{\perp} = \tilde{J}_0^{\perp}/\sqrt{2a}$ and $J_0^z = \tilde{J}_0^z/4$. However, due to the appearance of the cross term $\psi_1^{\dagger}(0) \psi_2(0) + \psi_2^{\dagger}(0) \psi_1(0)$ in (9), which describes the backward scattering of the magnetic

impurity on the conduction fermions, the problem becomes unsolvable. Therefore we have to find a method to eliminate this cross term. For this purpose, we define a new set of fermion operators

$$\begin{aligned}\Psi_1(x) &= \frac{1}{\sqrt{2}}(\psi_1(x) + \psi_2(-x)) \\ \Psi_2(x) &= \frac{1}{\sqrt{2}}(\psi_1(x) - \psi_2(-x))\end{aligned}\tag{10}$$

and the density operators $\tilde{\rho}_{1(2)}(x) = \frac{1}{L} \sum_p \tilde{\rho}_{1(2)}(p) e^{ipx}$ and $\tilde{\rho}_{1(2)}(p) = \sum_k \Psi_{1(2)}^+(k+p) \Psi_{1(2)}(k)$ which have the standard commutation relation of the right mover, $[\tilde{\rho}_{1(2)}(-q), \tilde{\rho}_{1(2)}(q')] = \frac{qL}{2\pi} \delta_{qq'}$, $[\tilde{\rho}_1(q), \tilde{\rho}_2(q')] = 0$. Now the bosonization representation of the fermion fields $\Psi_{1(2)}(x)$ can be performed in the standard way [21–23]

$$\begin{aligned}\Psi_{1(2)}(x) &= \frac{1}{\sqrt{2\pi\alpha}} e^{-i\phi_{1(2)}(x)} \\ \phi_{1(2)}(x) &= i \frac{2\pi}{L} \sum_p \frac{e^{-\frac{\alpha}{2}|p|-ipx}}{p} \tilde{\rho}_{1(2)}(p)\end{aligned}\tag{11}$$

where $\partial_x \phi_{1(2)}(x) = 2\pi \tilde{\rho}_{1(2)}(x)$. Using Eq. (10) to express the density operators $\rho_{1(2)}(x)$

$$\begin{aligned}\rho_1(x) &= \frac{1}{2} [\tilde{\rho}_1(x) + \tilde{\rho}_2(x) + \Psi_1^+(x) \Psi_2(x) + \Psi_2^+(x) \Psi_1(x)] \\ \rho_2(x) &= \frac{1}{2} [\tilde{\rho}_1(-x) + \tilde{\rho}_2(-x) - \Psi_1^+(-x) \Psi_2(-x) \\ &\quad - \Psi_2^+(-x) \Psi_1(-x)] \\ \psi_1^+(0) \psi_2(0) + \psi_2^+(0) \psi_1(0) &= \tilde{\rho}_1(0) - \tilde{\rho}_2(0)\end{aligned}\tag{12}$$

and defining new boson fields: $\phi_{\pm}(x) = \phi_1(x) \pm \phi_2(x)$, which satisfy the commutation relations: $[\phi'_{\pm}(x), \phi_{\pm}(y)] = i4\pi\delta(x-y)$, $[\phi'_+(x), \phi_-(y)] = 0$, the Hamiltonians (7) and (9) can be rewritten as

$$\begin{aligned}H &= \frac{v_F}{8\pi} \int dx \{ (\phi'_+(x))^2 + \gamma \phi'_+(x) \phi'_+(-x) \\ &\quad + (\phi'_-(x))^2 - \frac{4\gamma}{\alpha^2} \cos(\phi_-(x)) \cos(\phi_-(-x)) \}\end{aligned}\tag{13}$$

$$\begin{aligned}H_{im} &= \sqrt{2} J_0^{\perp} (\tilde{f}^+ \Psi_1(0) e^{iN(0)} + h.c.) \\ &\quad + \frac{J_0^z}{2\pi} (\tilde{f}^+ \tilde{f} - \frac{1}{2}) (\phi'_+(0) + \phi'_-(0))\end{aligned}\tag{14}$$

where $\phi'_\pm(x) \equiv \partial_x \phi_\pm(x)$. The cross term in Eq. (14) has vanished, but the boson field $\phi_-(x)$ in (13) becomes strongly self-interacting. The Hamiltonian (14) can be rewritten as

$$\begin{aligned} H_{im} = & \sqrt{2}J_0^\perp(\tilde{f}^+\Psi_1(0)e^{iN(0)} + h.c.) \\ & + \frac{J_0^z(k=0)}{2\pi}(\tilde{f}^+\tilde{f} - \frac{1}{2})\phi'_+(0) \\ & + \frac{J_0^z(k=2k_F)}{2\pi}(\tilde{f}^+\tilde{f} - \frac{1}{2})\phi'_-(0) \end{aligned} \quad (15)$$

where for simplicity, we use $J_0^z(k=0)$ and $J_0^z(k=2k_F)$ to indicate the forward and backward scattering interaction strength, respectively, because they show different effects on the system. We can cancel the J_0^z term in (15) by the following unitary transformation

$$U = \exp\{i(\tilde{f}^+\tilde{f} - \frac{1}{2})[\frac{g\delta_+}{\pi}\phi_+(0) + \frac{\delta_-}{\pi}\phi_-(0)]\} \quad (16)$$

where, $\delta_+ = \arctan(\frac{J_0^z(k=0)}{2v_F\sqrt{1-\gamma^2}})$, $\delta_- = \arctan(\frac{J_0^z(k=2k_F)}{2v_F})$, and $g = (\frac{1-\gamma}{1+\gamma})^{1/2}$ is a dimensionless coupling strength parameter.

However, the backward scattering potential has a drastical influence on the fermions $\Psi_{1(2)}(x)$, and induces the strong coupling between the fermion fields $\Psi_{1(2)}(x)$ and \tilde{f} at the impurity site $x = 0$. In the strong coupling limit induced by the backward scattering potential, i.e., the phase shift δ_- takes the value: $\delta_-^c = -\pi/2$, and taking the gauge transformations: $\Psi_{1(2)}(x) = \bar{\Psi}_{1(2)}(x)e^{i\theta_{1(2)}}$, $\theta_1 - \theta_2 = 2\delta_-(\tilde{f}^+\tilde{f} - 1/2)$, we can rewrite the total Hamiltonian as

$$\begin{aligned} \bar{H}_T = & U^\dagger(H + H_{im})U \\ = & \frac{v_F}{8\pi} \int dx \{(\bar{\phi}'_+(x))^2 + \gamma\bar{\phi}'_+(x)\bar{\phi}'_+(-x) \\ & + (\bar{\phi}'_-(x))^2 + \frac{4\gamma}{\alpha^2} \cos(\bar{\phi}_-(x)) \cos(\bar{\phi}_-(-x))\} \\ & + \frac{1}{\sqrt{\pi\alpha}} J_0^\perp(\tilde{f}^+ e^{-i(\frac{g\delta_+}{\pi} + \frac{1}{2})\bar{\phi}_+(0)} U^\dagger e^{iN(0)} U + h.c.) \end{aligned} \quad (17)$$

where $\bar{\phi}_\pm(x) = \bar{\phi}_1(x) \pm \bar{\phi}_2(x)$, $\partial_x \bar{\phi}_{1(2)}(x) = 2\pi\bar{\rho}_{1(2)}(x)$, $\bar{\rho}_{1(2)}(x) = \bar{\Psi}_{1(2)}^\dagger(x)\bar{\Psi}_{1(2)}(x)$. The last term can be easily obtained by performing the unitary transformation $U^\dagger \tilde{f}^+ U$ which contributes the phase factor $-ig\delta_+ / (\pi)\bar{\phi}_+(0) - i\delta_- / (\pi)\bar{\phi}_-(0)$. In the strong coupling limit ($\delta_-^c = -\pi/2$), by using Eq.(11), we can obtain the last term in (17). If we redefine the following new fields:

$$\begin{aligned}
\bar{\psi}_1(x) &= \frac{1}{\sqrt{2}}(\bar{\Psi}_1(x) + \bar{\Psi}_2(x)) \\
\bar{\psi}_2(x) &= \frac{1}{\sqrt{2}}(\bar{\Psi}_1(-x) - \bar{\Psi}_2(-x))
\end{aligned} \tag{18}$$

where the bosonization representation of the fermion fields $\bar{\psi}_{1(2)}(x)$ is $\bar{\psi}_{1(2)}(x) = (\frac{1}{2\pi\alpha})^{1/2} e^{-i\bar{\Phi}_{1(2)}(x)}$, the total Hamiltonian can be written as

$$\begin{aligned}
H_T^c &= \frac{v_F}{4\pi} \int dx \{ (\bar{\Phi}'_1(x))^2 + \gamma \bar{\Phi}'_1(x) \bar{\Phi}'_1(-x) \\
&\quad + (\bar{\Phi}'_2(x))^2 + \gamma \bar{\Phi}'_2(x) \bar{\Phi}'_2(-x) \\
&\quad + \sqrt{2} J_0^\perp [\tilde{f}^+ \psi(0) + \psi^+(0) \tilde{f}] \}
\end{aligned} \tag{19}$$

where $\psi(0) = (\frac{1}{2\pi\alpha})^{1/2} \exp\{-i(\frac{1}{2} + \frac{g\delta_+}{\pi})(\bar{\Phi}_1(0) + \bar{\Phi}_2(0))\}$. After these transformations, the phase factor $N(0)$ disappears in the total Hamiltonian (19). If the dimensionless coupling strength parameter g takes the values, $g \geq 1$, in the strong coupling limit induced by the forward scattering potential, the phase shift δ_+ satisfies the relation: $\delta_+^c = -\pi/(2g)$, $\psi(0)$ becomes a constant field, the total Hamiltonian (19) is very similar to that in Ref. [24] derived from the quantum dot. The J_0^\perp -term opens a gap in the energy spectrum of the fermion \tilde{f} , therefore, in the low temperature and low energy limit, the Green's function of the fermion \tilde{f} is an exponential decaying function, the impurity susceptibility exponentially goes to zero as the temperature going to zero. However, if the dimensionless coupling strength parameter g is less than one, $g < 1$, the phase shift δ_+ only takes the value: $\delta_+^c = -\pi/2$, the temperature dependence of the impurity susceptibility has a power-law form (see below). Near the strong coupling critical point induced by the forward and backward scattering potentials, we have the leading irrelevant Hamiltonian

$$\Delta H = \lambda(\tilde{f}^+ \tilde{f} - \frac{1}{2})\phi'_+(0) + \lambda'(\tilde{f}^+ \tilde{f} - \frac{1}{2})\phi'_-(0) \tag{20}$$

where $\lambda = -v_F(\delta_+ - \delta_+^c)/\pi$ and $\lambda' = -v_F(\delta_- - \delta_-^c)/\pi$.

According to Eq. (10), we have following relations at the point $x = 0$:

$$\begin{aligned}
\rho_1(0) + \rho_2(0) &= \tilde{\rho}_1(0) + \tilde{\rho}_2(0) = \frac{1}{2\pi}\phi'_+(0) \\
\rho_1(0) - \rho_2(0) &= \frac{1}{\pi\alpha} \cos(\phi_-(0)) \\
\psi_1^+(0)\psi_2(0) + \psi_2^+(0)\psi_1(0) &= \frac{1}{2\pi}\phi'_-(0)
\end{aligned} \tag{21}$$

Therefore, in the strong coupling limit, from (18), (19) and (21), we can easily obtain the following correlation functions

$$\begin{aligned}
\langle \phi'_-(0,0)\phi'_-(0,t) \rangle &\sim \left(\frac{1}{t}\right)^{\frac{2}{g}} \\
\langle e^{i\phi_+(0,0)}e^{-i\phi_+(0,t)} \rangle &\sim \left(\frac{1}{t}\right)^{\frac{2}{g}} \\
\langle \phi'_+(0,0)\phi'_+(0,t) \rangle &\sim \left(\frac{1}{t}\right)^2
\end{aligned} \tag{22}$$

For an antiferromagnetic Heisenberg chain, the exponent g equals $1/2$ (γ is very large).

From Eq.(19) we can obtain the Green's function of the impurity fermion

$$G_{\tilde{f}}(t) = \langle \tilde{f}(0)\tilde{f}^+(t) \rangle \sim \begin{cases} 0, & g \geq 1 \\ \left(\frac{1}{t}\right)^{2-\frac{1}{2g}(1-g)^2}, & g_c < g < 1 \\ e^{-i\epsilon_f t}, & g \leq g_c \end{cases} \tag{23}$$

where ϵ_f is the level of the impurity fermion f , and g_c is defined as that: $4g_c = (1 - g_c)^2$. The physical interpretation of the special parameter g_c is that at this point the self-energy of the impurity fermion \tilde{f} induced by the interaction term in (19) has a linear frequency dependence. With the help of Eq.(22) we see that in Eq.(20) for the antiferromagnetic case $g < 1$, the λ -term is dominant while for the ferromagnetic case $g > 1$, the λ' -term is dominant. However, in the case of $g > 1$, the Green's function of the fermion f is exponentially decaying (in Eq.(23) we take it as zero in the long time limit), only the λ -term be relevant. By using Eqs. (22), and (23), the correlation function of ΔH (Eq. (20)) can be written as $\langle (\tilde{f}^+\tilde{f} - 1/2)(t) \cdot (\tilde{f}^+\tilde{f} - 1/2)(0) \rangle \sim \langle \phi'_+(0,t)\phi'_+(0,0) \rangle$, if we omit the vacuum fluctuation of the fermion \tilde{f} , it reads

$$\langle \Delta H(0,0)\Delta H(0,t) \rangle \sim \begin{cases} \lambda^2\left(\frac{1}{t}\right)^{6-\frac{1}{g}(1-g)^2}, & g_c < g < 1 \\ \lambda^2\left(\frac{1}{t}\right)^2, & g \leq g_c \end{cases} \tag{24}$$

while the spin susceptibility of the impurity is (omitting the vacuum fluctuation of the fermion \tilde{f})

$$\langle S^z(0)S^z(t) \rangle \sim \left(\frac{1}{t}\right)^{4-\frac{1}{g}(1-g)^2}, \quad g_c < g < 1 \tag{25}$$

From Eqs. (24) and (25), using the relation between specific heat and free energy (Eq.(24) gives the impurity free energy), we can easily obtain the temperature dependence of the impurity specific heat $C_{im}(T)$ and spin susceptibility $\chi_{im}(T)$

$$C_{im}(T) \sim \begin{cases} \lambda^2 T^{4-\frac{1}{g}(1-g)^2}, & g_c < g < 1 \\ constant, & g \leq g_c \end{cases} \quad (26)$$

$$\chi_{im}(T) \sim \begin{cases} T^{3-\frac{1}{g}(1-g)^2}, & g_c < g < 1 \\ T^{-1}, & g \leq g_c \end{cases} \quad (27)$$

which show an unusual temperature dependence in the low energy and low temperature limit. We can give a brief explanation about present results. It is worth noted that because we use an usual Kondo interaction as in [21] which is different from that in [20], we obtain different results from that in [20]. The symmetry of the Kondo interaction term significantly influences the low temperature behavior of the magnetic impurity. In generally, the Heisenberg chain can be described by an interacting spinless electron system. As the interactions among the electrons are repulsive, the bound state of the conduction electron and the impurity fermion is weakened by the repulsive interaction of the conduction electrons. For an enough strongly repulsive interaction $g = g_0$, where g_0 satisfies: $3g_0 - (1 - g_0)^2 = 0$, the bound state at the impurity site is broken, and the impurity fermion begins to show a free-type behavior. For an antiferromagnetic Heisenberg chain, the dimensionless coupling strength parameter takes the value [21] $g = \frac{1}{2}$, the impurity specific heat $C_{im}(T)$ is proportional to $T^{7/2}$, and the temperature dependence of the impurity spin susceptibility $\chi_{im}(T)$ is $\chi_{im}(T) \sim T^{5/2}$. Because in the Jordan-Wigner representation, the Heisenberg chain can be reduced into a very simple form, these anomalous temperature dependences of the impurity specific heat and spin susceptibility should be easily observed in the numerical calculations. Although these temperature dependences heavily rely upon the dimensionless coupling strength parameter g , we still have a temperature independent Wilson ratio which exists in the high-dimensional Kondo problems.

In summary, by using the bosonization method, we have studied in detail the low temperature physical behavior of the spin-1/2 magnetic impurity in Heisenberg chain, and shown that the impurity specific heat and spin susceptibility have unusual temperature dependence behavior in the low energy and low temperature limit.

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